

**PROCEEDINGS
OF
THE JAPAN-U.S. WORKSHOP ON
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LANDSLIDE, DEBRIS FLOW
PREDICTION AND CONTROL**

September 30 - October 2, 1991
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ORIGIN OF STEP-POOL SEQUENCES IN HIGH GRADIENT STREAMS:
A FLUME EXPERIMENT

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SUMMARY: A series of flume experiments was undertaken to determine the domain of flow conditions under which step-pool sequences are formed. This domain can be characterized by the ratio of average shear stress to the shear stress required to move the largest grain sizes (τ/τ_{cr}) and the Froude number (Fr). Step spacing was correlated with the antidune wavelength and steps did not form when particle motion was continuous under high sediment transport rates.

1. INTRODUCTION

Many high gradient, boulder-bed streams are characterized by alternating steep and gentle segments which typically create a staircase structure of steps and pools. This structure is commonly expressed at two distinct scales (Fig. 1). The first is channel-spanning steps composed of large boulders interspersed with small pools less than one channel width in length; the step and its associated pool is termed a step-pool sequence. Multiple step-pool sequences alternating with large pools longer than the channel width are termed channel units (Grant *et al.*, 1990) or swells (Kishi *et al.*, 1987).

Channel units and individual step-pools are important from several perspectives. These structures constitute a major component of flow resistance in mountain streams and dissipate stream energy that might otherwise be available for sediment transport and channel erosion (Ashida *et al.*, 1976; 1986 a,b). Sediment transport and storage in mountain

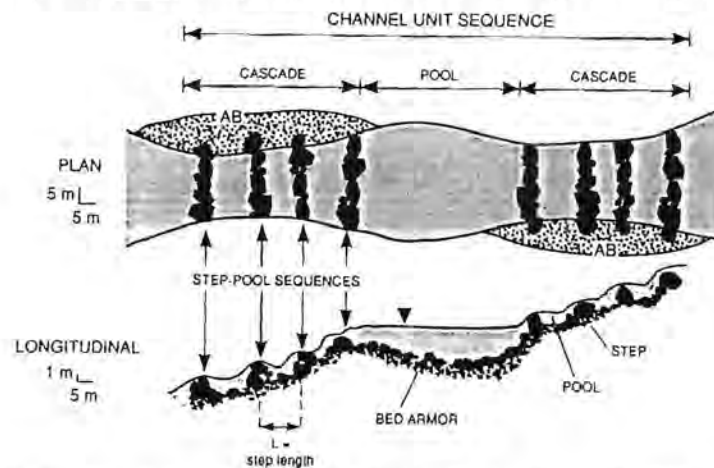


Figure 1: Plan and side view of channel unit and step-pool sequences, showing bed armor and alternate bars (AB).

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streams are regulated by the geometry of step-pool sequences; bedload transport during low to moderate discharges involves movement of relatively fine material (gravel and sand) through steep channel units and deposition in pools (Whittaker, 1987 a,b; Sawada *et al.*, 1983). Complex hydraulics generated by alternating steep and gentle gradient units provide diverse habitat for stream organisms, notably fish, which show strong preferences for specific channel units (Gregory *et al.*, 1991). Maintaining channel unit and step-pool morphology is thus important to minimize channel instability, reduce sediment transport, and maintain stream health for aquatic ecosystems.

Little is known, however, about the mechanisms and flow conditions required for step-pool formation. Step-pools generally are stable during most flows but can be reworked or destroyed by large floods and debris flows (Hayward, 1980; Sawada *et al.*, 1983; Ashida *et al.*, 1981). Because recurrence intervals for flows forming channel units are likely to be 20 to 50 years or greater (Grant *et al.*, 1990), field data on step-pool formation are sparse and limited to visual observations of recovery of step-pool topography following debris flow (Sawada *et al.*, 1983). Most research on step-pool morphology has been done in flumes and much of this important work is known only in Japan (Judd and Peterson, 1969; Whittaker and Jaeggi, 1982; Ashida *et al.*, 1984; 1985; 1986 a,b; Hasegawa, 1988).

In this paper we report on a series of flume experiments which examined the basic processes of step-pool formation under a range of flow and sediment transport conditions. These experiments included both clear water flows and runs where sediment was supplied from upstream. The latter is a more realistic representation of natural river systems, since step-pools form during large floods when bedload transport rates are high.

2. METHODS

A series of flume experiments were conducted in fall, 1988 in the 11 m long by 0.5 m wide adjustable-slope flume located at the Public Works Research Institute in Tsukuba, Japan. Water was delivered to the flume by a pump with a maximum capacity of 50 l/sec; discharges were measured at the lower end of the flume through a calibrated V-notch weir. Unimodal bedload mixtures, with maximum grain sizes equal to 64 and 30 mm, were used as bed material (Fig. 2) and supplied at constant rates during some runs by a belt-conveyor feed system located at the upstream end of the flume. When the finer mixture was used, the flume width was reduced to 0.25 m to maintain a constant ratio of maximum grain size diameter to channel width.

Before each run, slope was adjusted, and wetted bed material was added to a uniform depth of 10 cm along the entire length of the flume. Water surface and bed material slope were then allowed to reach equilibrium with the imposed water and (in some runs) sediment discharge. Water depth, bed elevation, and velocity were measured at 7 minute intervals using point gauges and micro velocity meters placed at 0.5 m stations along the flume; three readings equidistant across the flume width were taken at each station. Bedload discharged at the flume outlet was collected for 30-seconds at 1-min intervals throughout the run. The collected bedload was weighed and volumes determined, and the diameter of the intermediate (b) axis of the 10 largest particles caught was measured. After each run, the bed was examined and the location, orientation with respect to flume sidewalls, and percent of channel spanned by each step formed during the run were measured. Beds were photographed and bed material taken for particle size analysis for some runs.

Total boundary shear stress for each station τ_s was calculated as:

$$\tau_s = \rho g d \sin \theta_s \quad (1)$$

where ρ is the density of water (1.0 g/cm³), g is the gravitational constant, d is the depth of water, and θ_s is the local bed slope averaged over 0.5m up- and downstream from

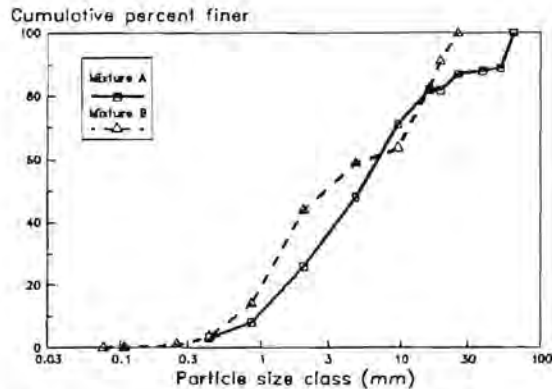


Figure 2: Particle size distributions for the two experimental mixtures.

the station. The Froude number (FR_s) for each station was determined as:

$$FR_s = \frac{v_s}{\sqrt{gd}} \quad (2)$$

where v_s is the average measured velocity for each station. Average shear stress for the run (τ_r) was determined by averaging all τ_s values and average Froude number for the run (FR_r) was similarly calculated by averaging all FR_s values.

3. EXPERIMENTAL RESULTS

The experiments were carried out in three parts. We conducted preliminary runs to characterize the relationship between shear stress and size of particle entrained for the two mixtures. Next, we varied slope and discharge in a series of clear-water (no sediment added) runs to define the range of flow conditions required to form step-pools. Finally, we fed sediment into the upstream end of the flume to investigate the effects of high sediment transport rates on step-pool formation.

3.1 Incipient motion experiments

A series of incipient motion experiments were conducted with both bedload mixtures to determine critical shear stresses for entraining bed material. Previous field work had determined that steps are composed of the largest bed particles in the stream (Grant *et al.*, 1990), so results were expressed by the average run shear stress τ_r and the largest particles captured during the run (Fig. 3). Results from these experiments were used in later runs to define a dimensionless shear stress τ^* :

$$\tau^* = \frac{\tau_r}{\tau_{cr}} \quad (3)$$

where τ_{cr} is the critical shear stress for entraining the largest particles in the mixture (330 dynes/cm² for mixture A and 170 dynes/cm² for mixture B).

3.2 Mechanisms and flow conditions for step formation

Step-pools were observed in the bed of the flume following some runs. Steps were distinguished as a linear arrangement of imbricated particles oriented more or less

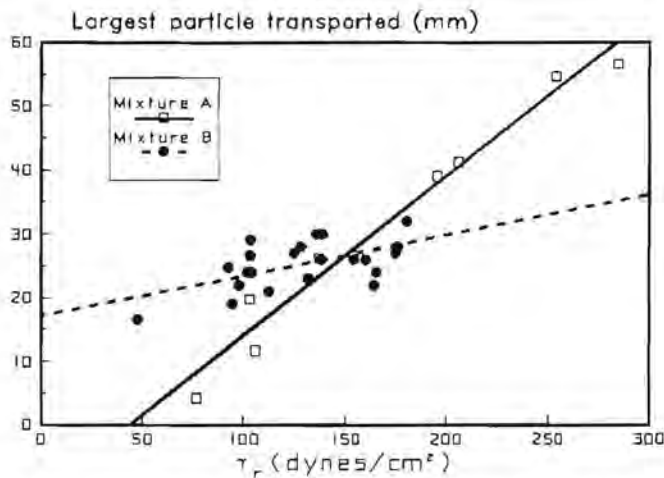


Figure 3: Results from incipient motion experiments for the two mixtures shown in Fig. 2.

perpendicular to flow (Fig. 4). Pools were more difficult to observe but appeared after the flume was drained as pockets of residual water, often partially filled with fines, located just downstream of steps. Reducing flow depth to less than the diameter of the largest grains following a run assisted recognition of steps and pools. Because the height of individual steps was the same as the grain roughness, it was not possible to distinguish steps by longitudinal bed profiles. Instead a visual rating system was employed to evaluate the degree of step development. The rating system went from 1 (very poor or obscure step, no clear imbrication, particles aligned obliquely to flow, no clear pool) to 5 (very well-developed step, particles clearly imbricated, step oriented perpendicular to flow, well-defined pool downstream). Steps with a rating of 3 or greater were termed 'well-developed' steps while steps with a rating of 1 or 2 were termed 'obscure' steps.

In runs where step-pools formed, step formation could be observed through the clear sidewall of the flume. The process we observed is similar to that described by Whittaker and Jaeggi (1983) and Ashida *et al.* (1984) with some modification. Initially, particle transport rates were high and the entire bed was active with rolling, sliding, and saltating grains. Within 1-2 minutes after the beginning of the run, regular antidunes formed on the water surface, resulting in bed deformation in phase with the water surface. As the bed deformed, individual large particles would intermittently come to rest under or immediately downstream of the antidune crest. These large particles trapped other smaller particles, creating a cluster of imbricated grains. The shallower depth over these clusters caused the formation of a hydraulic jump in the antidune trough. The turbulence associated with this jump scoured the bed immediately downstream of the stalled grains, accentuating the relief of the step. This entire process occurred simultaneously with development of an armor or coarse surface layer over the entire bed. Infilling of the scour hole with fines occurred when the water level dropped at the end of the run reducing the strength of the hydraulic jump.

As predicted by these observations, there was a good correlation ($r^2 = 0.50$) between the interstep spacing (measured from step crest to crest) and the antidune wavelength, defined by the wave number $L = 2\pi v_r^2/g$, where v_r is the mean velocity for the run (Kennedy, 1963) (Fig. 5). Some scatter is probably due to using the flume as opposed to local (station) averages when calculating spacing and velocity. These results suggest that step spacing might be a useful field indicator of paleovelocities required for particle entrainment and step formation.



Figure 4: An example of a step-pool. Note the imbricated particles in the step and fines filling the downstream area (pool). Flow is from right to left. Scale is graduated in centimeters.

The domain of step formation can be characterized by two dimensionless parameters: the ratio of average shear stress to the shear stress required to move the largest grain sizes (τ^*) and the average run Froude number (Fr_r). Flow conditions for formation of well-developed steps were defined reasonably well in our experiments by the range $0.5 < \tau^* \leq 1.0$ and $0.7 < Fr_r \leq 1.0$ (Fig. 6). For comparison, data replotted from Ashida *et al* (1984) is presented. They show steps forming over the same ranges of τ^* but at Froude numbers approximately twice as high as what we observed (Fig. 6). Reasons for this discrepancy are unclear but may have to do with differences in how the Froude number was calculated as Ashida *et al* used the shear velocity rather than Equation 2. There may also be differences in criteria used to define steps. Only four of our runs had average Froude numbers greater than unity and we observed steps forming in all of them, so steps may continue to form under supercritical flow conditions, as suggested by Ashida *et al*. In both studies, there is common agreement that the critical conditions for step-pool formation are: 1) heterogeneous bed mixture; 2) critical shear stress for the bed material exceeded but less than critical shear stress for the maximum grain size; 3) Froude numbers near unity and formation of antidunes.

3.3 Step formation during sediment transport

Several runs were conducted to examine the effect of high sediment transport rates on step formation. We used an initial set of hydraulic conditions that had produced steps in previous clear-water runs. In addition, we fed sediment (mixture B) into the upstream end of the flume at what we calculated to be an equilibrium transport rate, using the method of Bathurst (1987). In the example reported here, the discharge was 4 l/s, initial bed slope was 0.04, and sediment was fed at a rate of 1,140 cm³/min. The flow conditions corresponded to an initial $\tau^* = 0.7$ and $Fr = 0.8$, well within the region of step formation (Fig. 6). The experiment was conducted in three parts (Fig. 7). First, steps were allowed to form under a clear water regime (0-20 minutes). After the flume was drained to permit observation of the bed, both water and sediment were fed at constant rates for 130 minutes, interrupted only at 90 minutes to observe the bed. Sediment feed was stopped at 150 minutes and clear water allowed to run for an additional 30 minutes. Sediment outflow and grain size distribution of the bedload exported from the flume were measured throughout the experiment, as was the grain size distribution of the initial and final bed.

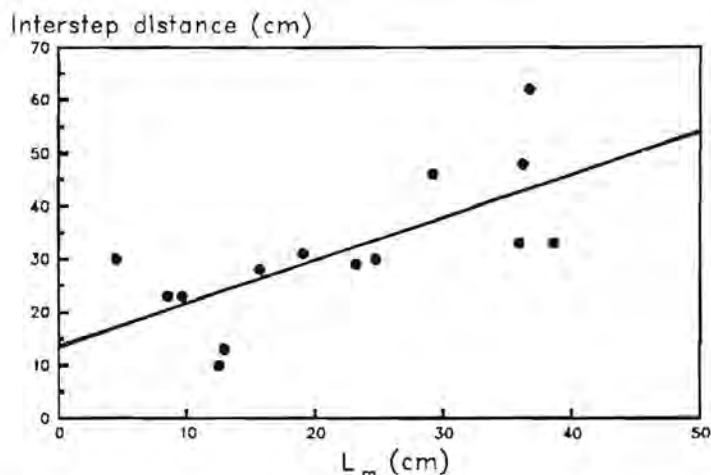


Figure 5: Interstep distance (measured crest-to-crest) as a function of antidune spacing as defined by the wave number $L_m = 2\pi v^2/g$.

Results from this experiment demonstrate the interactions among development of steps, bed armor, and alternate bars. During the initial 20 minute period when no sediment was introduced, well-developed steps formed and the bed armored in the manner previously described (Fig. 7). Alternate bars did not form during this period and steps spanned the entire flume width. Both the sediment transport rate and the maximum size of transported particles were initially quite high but dropped markedly as the bed armored. Armoring of the bed proceeded from the upper to the lower part of the flume, suggesting that absence of upstream supply was a key factor contributing to armor formation, as has been suggested elsewhere (Dietrich et al, 1989).

A different set of bedforms and sediment transport processes was observed during the 130 minute period when sediment was fed into the flume. As it turns out, the sediment input and outflow rates were not in equilibrium during the experiment; average transport rate from 20 - 150 minutes was 515 cm³/min, roughly half of the feed rate. This resulted in significant aggradation in the upper end of the flume. The slope steepened from 0.037 at 12 minutes to 0.053 at 146 minutes. Because of the increased gradient, Fr_c increased from 0.78 to 0.93 and τ_c increased from 0.73 to 0.94 during this period. These increases were still within the domain of step formation (Fig. 6), so steps should have formed. Only poorly-formed steps (rating number ≤ 2) were observed, however, after either 90 or 150 minutes when the flume was drained. Instead, well-developed alternate bars emerged parallel to the walls along both sides of the flume.

Step formation was inhibited by high sediment transport rates and lack of bed armoring. Several factors contributed to this. High transport rates resulted in frequent grain-to-grain collisions so individual particles never stopped for very long. Ballistics frequently disrupted accumulating clusters of particles so the local conditions for hydraulic jump and step formation occurred only occasionally. A second factor was the longitudinal segregation of bedload into coarse and fine zones. This phenomenon, described by Iseya and Ikeda (1987), was observed in both the size of largest particles in transport (Fig. 7) and the grain size distribution of the bedload (Fig. 8). From 20 - 90 minutes, bedload was predominantly fine sediment while from 90 - 140 minutes, the bedload was significantly coarser than the initial bed material and sediment feed (Fig. 8). Another fine sediment zone was beginning to appear at the flume outlet at the time sediment feed was stopped at 150 minutes (Fig. 8). The mechanism by which this segregation inhibited step formation depended on whether bedload was either fine or coarse sediment. In areas

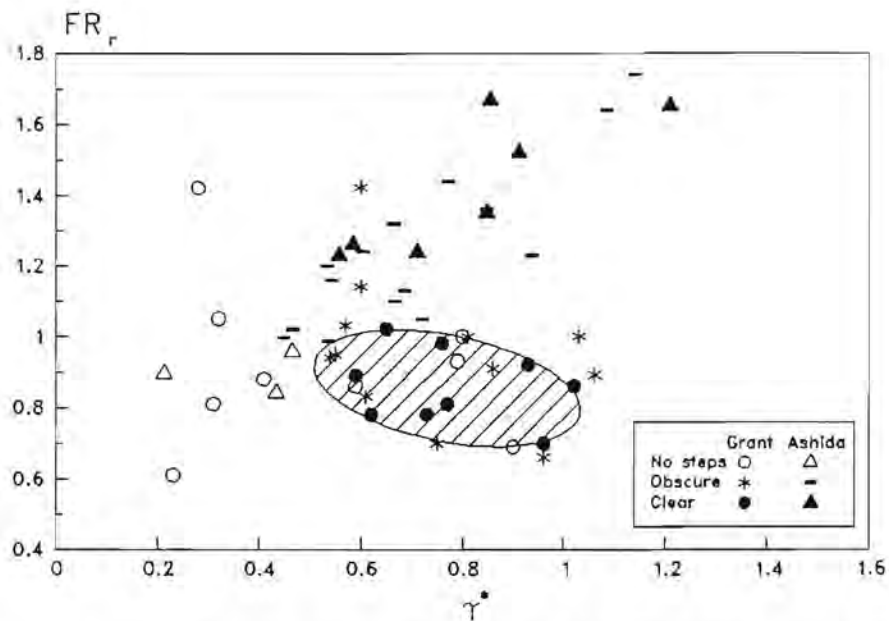


Figure 6: Domain of step formation. Shaded area represents zone of clear steps. Data from Ashida et al (1984) also shown.

of the bed dominated by fine sediment, individual larger grains rarely stopped because they protruded higher in the water column and were thus exposed to higher shear stresses. In addition, fine sediment filled the interstices between larger grains, thereby smoothing the bed and reducing flow resistance (Iseya and Ikeda, 1987; Whiting et al, 1988). In regions dominated by coarse sediment, grain-to-grain collisions, as described above, were more frequent. In neither case were well-developed steps observed.

Steps did reform, however, when water alone was allowed to run for 30 minutes after sediment feed was stopped at 150 minutes (Fig. 7). Step formation was accompanied by formation of a bed armor layer (Fig. 8). Most steps formed adjacent to alternate bars where zones of higher shear stress were concentrated; these steps only spanned the wetted channel and could not be traced onto the surface of the adjacent bar. The constricted width adjacent to bars favored antidune formation which may have promoted forming steps.

4. Discussion and summary

Taken with the earlier work of Whittaker and Jaeggi (1983) and Ashida et al (1984), these experiments provide the basis for understanding the mechanisms and flow conditions for step-pool formation. Widely-sorted bed material is required for two reasons. First, so large grains which protrude into the flow column can produce hydraulic jumps and second, so there is sufficient disparity in entrainment thresholds and bed roughness that larger grains can trap smaller ones. Sorting coefficients for the bed mixtures, defined by the ratio D_{94}/D_{16} , were 17 and 15 for the A and B mixtures, respectively (Fig. 2); sorting coefficients of 20 or greater have been observed in step-pool streams in the field (Grant et al, 1990). Bedforms somewhat analogous to steps, such as transverse ribs, may be produced by a process similar to step formation where bed material is more uniformly sorted (McDonald and Bannerjee, 1971; McDonald and Day, 1978; Koster, 1978; Kishi et al, 1987).

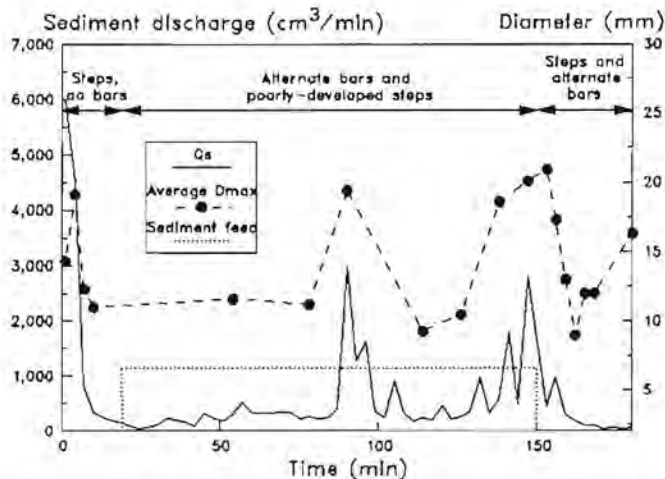


Figure 7: Sediment feed, bedload discharge, and average diameter of largest 10 particles caught at flume outlet during run 22. Bedforms observed during the run are noted.

Flow conditions for step formation require near-critical to supercritical flow conditions over the bed and must be close to but not exceed the entrainment thresholds for the largest (D_{90} or larger) particles. Paleohydraulic calculations from field observations give similar estimates of Froude numbers for these entrainment thresholds (Grant *et al.*, 1990). A further constraint on step-pool formation is that the relative roughness (ratio of particle size to flow depth) be close to unity. Otherwise protruding particles cannot generate hydraulic jumps. This conclusion has also been confirmed by field observations and paleohydraulic calculations (Bowman, 1977; Jarrett and Costa 1986; Grant *et al.*, 1990).

While step-pool formation requires that critical shear stresses be exceeded for most of the bed material, our results also indicate that sediment transport rates cannot be too high. This suggests why step-pools are typically found only in streams where sediment

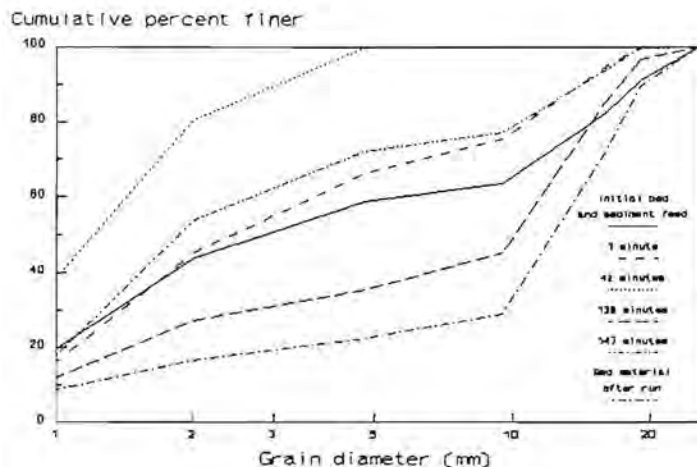


Figure 8: Particle size distributions for bedload and bed material during run 22. Times correspond to those shown in Fig. 7.

supply rates are relatively low and canyons are narrow, such as steeply dissected channels draining areas with competent bedrock. Step-pools are not common in wide alluvial channels with abundant sediment supply -- glacial outwash streams, for example -- both because the bed material tends to be more uniform and sediment transport rates are high (e.g. Fahnestock, 1963).

Factors controlling formation of step-pools appear to be fundamentally different than those creating large pools. While step-pools are features whose horizontal and vertical dimensions are scaled by the depth of flow and coarsest grain sizes, large pools have lengths scaled by the channel width (Fig. 1). These larger features owe their origin to secondary circulation patterns developed around channel curvature or resistant boundary material (Lisle, 1986).

Given the important hydraulic, geomorphic, and ecologic roles that step-pools play in stream and riparian systems, we must recognize conditions which favor step formation. Effective strategies for restoring ecologic functioning in streams following disturbances such as debris flows or large floods may involve promoting formation of step-pool sequences. This may include modifying channel geometry -- channel width, for example -- to create high Froude numbers and shear stresses during high flow events, introducing large particles or resistant boundary material, or in the case of regulated streams, by altering the flow regime to produce the required flow conditions, at least temporarily. Future research should evaluate the effectiveness of these strategies in a range of channel environments.

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